حل لبعض الاسئلة المختارة من شابتر 2

ملاحظة: الحل عبارة عن صور وليس نص مكتوب. واذا لم يوجد رقم السؤال فوق الصورة، فتكون الصورة(الحل) مكملة للصورة(للحل) السابقة. على سبيل المثال سؤال رقم 39 من سكشن 1، له صورتان.

Ch 2.1

2.1.13

The truth table for $\square (p \wedge q) \vee (p \vee q)$ is

p	q	$p \wedge q$	$\square (p \wedge q)$	$p \vee q$	$\square (p \wedge q) \vee (p \vee q)$
T	T	T	F	T	T
T	F	F	T	T	T
F	T	F	T	T	T
F	F	F	T	F	T

2.1.19

Consider the following statement forms:

$p \wedge \mathbf{t}$ and p.

The objective is to determine whether the above statement forms are logically equivalent or not, also construct a truth table and also a sentence justifying the answer.

Comment

Step 2 of 2

Two statements forms are said to logically equivalent if, and only if, they always have the same truth values.

Here, $\mathbf{t}$ stands for tautology whose truth value is always true.

The truth table for the given statement forms is as follows:

p	$\mathbf{t}$	$p \wedge \mathbf{t}$
T	T	T
F	T	F

$p \wedge \mathbf{t}$ and p always have same truth values, so they are logically equivalent.

$\therefore$ From the truth table, $p \wedge \mathbf{t}$ and p have the same truth values, so they are **logically equivalent**.

Ch 2.1

2.1.14

(a)

The objective is to write the argument using letters.

Let p represents n is divisible by 6, q represents n divisible by 3, and r represents the sum of the digits of n is divisible by 3.

Rewrite the statements using the letters.

If p then q.

If q then r.

Therefore, if p then r.

Comment

Step 2 of 2

(b)

The objective is to fill in the blanks for the given argument that has the same logical form as in part (a).

In the given argument in the third sentence, p is the statement between "if – then".

Thus, p is the statement "this function is polynomial".

In the first sentence, q is the statement following then.

Thus, q is the statement "this function is differentiable".

In the second sentence, r is the statement following then.

Thus, r is the statement "this function is continuous".

Replace p, q, r in the logical form then the argument is:

If this function is a polynomial, then this function is differentiable.

If this function is differentiable, then this function is continuous.

Therefore, if this function is a polynomial, then this function is continuous

Ch 2.1

2.1.15

a)

Given is a sentence "1,024 is the smallest 4-digit number that is a perfect square", we need to determine whether it is a statement or not.

This sentence is a statement because it fact stated that 1024 is the smallest four digit number which is a perfect square is true.

Comment

Step 2 of 4

b)

Given is a sentence "She is a mathematics major", we need to determine whether it is a statement or not.

This sentence is not a statement as its truthfulness or falseness depends on the person it is being referred to which is cannot be known through this sentence.

Comment

Step 3 of 4

c)

Given is a sentence " $128 = 2^6$ ", we need to determine whether it is a statement or not.

This sentence is a statement because it can be easily verified that this statement is true. There is no scope for ambiguity in this sentence.

Comment

Step 4 of 4

d)

Given is a sentence " $x = 2^6$ ", we need to determine whether it is a statement or not.

This sentence is not a statement because it cannot be verified whether the sentence is true or false as it depends upon the value of which is unknown.

Ch 2.1

2.1.24

Consider the pair of statement forms,

$(p \vee q) \vee (p \wedge r)$ **and** $(p \vee q) \wedge r$

The objective is to determine whether this pair of statement forms are logically equivalent or not. Justify the answer with truth tables and include a few words of explanation.

Comment

Step 2 of 3

Here tautology is indicated as t, and contradiction is indicated as c.

The truth table that shows the statement forms:

p	q	r	$p \vee q$	$p \wedge r$	$(p \vee q) \vee (p \wedge r)$	$(p \vee q) \wedge r$
T	T	T	T	T	T	T
T	T	F	T	F	T	F
T	F	T	T	T	T	T
T	F	F	T	F	T	F
F	T	T	T	F	T	T
F	T	F	T	F	T	F
F	F	T	F	F	F	F
F	F	F	F	F	F	F

Comment

Step 3 of 3

From the above truth table, the statement forms $(p \vee q) \vee (p \wedge r)$ **and** $(p \vee q) \wedge r$ have different truth values.

Hence, the statement forms $(p \vee q) \vee (p \wedge r)$ **and** $(p \vee q) \wedge r$ are not logically equivalent.

Ch 2.1

2.1.29

The objective is to write negation for the following statement using De Morgan's laws:

This computer program has a logical error in the first ten lines or it is being run with an incomplete data set.

Let p represents "this computer has a logical error in the first ten lines", and q stands for "this program is being run with an incomplete data set".

The symbolic notation for the statement is:

$p \vee q.$

Comment

Step 2 of 2

Take the negation of $p \vee q$:

$\sim(p \vee q).$

Use De Morgan's Law for $\sim(p \vee q).$

$\sim(p \vee q) \equiv \sim p \wedge \sim q.$

The negation of p is: The program does not have a logical error in the first ten lines and q is: this program is not being run with an incomplete data set.

The two statements are connected with $\wedge$ a conjunction which is referred as an "**and**".

The negation of the statement can be written as:

"This computer program does not have a logical error in the first ten lines and is not run with an incomplete data set."

Ch 2.1

2.1.32

Consider the following statement:

$-2 < x < 7$, for some real number x.

The objective is to write the negation of the above statement using De Morgan's law.

Comment

Step 2 of 2

Recall the following De Morgan's Law:

$\sim(p \wedge q) \equiv \sim p \vee \sim q$ for some statements p and q.

The given statement is equivalent to the form of $(p \wedge q)$.

$-2 < x$ and $x < 7$

Here, the statements p and q are as follows:

$p : -2 < x$
$q : x < 7$

Thus, the negation of the statement: $(p \wedge q)$ is, $\sim(p \wedge q) \equiv \sim p \vee \sim q$.

Write the negations of p and q as,

$\sim p : -2 \geq x$
$\sim q : x \geq 7$

Therefore, the negation of the given statement is, $\boxed{-2 \geq x \text{ or } x \geq 7}$.

Ch 2.1

2.1.37

The given statement is equivalent to

$0 > x$ and $x \geq -7$

Fro De Morgan's law, the negation is

$0 \not> x$ or $x \not\geq -7$

Which is equivalent to $0 \leq x$ or $x < -7$

2.1.39

The objective is to write negations for the following statement.

$$(num\_orders < 50 \text{ and } num\_instock > 300) \text{ or } (50 \leq num\_orders < 75 \text{ and } num\_instock > 500).$$

Take the negation of the statement.

$$\sim\left[\begin{array}{l}(num\_orders < 50 \text{ and } num\_instock > 300) \text{ or } \\ \quad (50 \leq num\_orders < 75 \text{ and } num\_instock > 500)\end{array}\right].$$

Replace and by $\wedge$ and or by $\vee$.

$$\equiv\sim\left[\begin{array}{l}(num\_orders < 50 \wedge num\_instock > 300) \vee \\ \quad (50 \leq num\_orders < 75 \wedge num\_instock > 500)\end{array}\right].$$

Comment

Step 2 of 4

Apply De Morgan's law to the statement.

$$\equiv\sim(num\_orders < 50 \wedge num\_instock > 300) \wedge \sim(50 \leq num\_orders < 75 \wedge num\_instock > 500).$$

Again, apply De Morgan's law to simplify the statement.

$$\equiv[\sim(num\_orders < 50) \vee \sim(num\_instock > 300)] \wedge [\sim(50 \leq num\_orders < 75) \vee \sim(num\_instock > 500)]$$

$$\equiv[\sim(num\_orders < 50) \vee \sim(num\_instock > 300)] \wedge [\sim(num\_orders \geq 50 \text{ and } num\_orders < 75) \vee \sim(num\_instock > 500)]$$

Ch 2.1

Replace and by $\wedge$.

$$\equiv [\sim(num\_orders < 50) \vee \sim(num\_instock > 300)] \wedge$$
$$[\sim(num\_orders \geq 50 \wedge num\_orders < 75) \vee \sim(num\_instock > 500)]$$

Apply De Morgan's law to the statement.

$$\equiv [\sim(num\_orders < 50) \vee \sim(num\_instock > 300)] \wedge$$
$$[[\sim(num\_orders \geq 50) \vee \sim(num\_orders < 75)] \vee \sim(num\_instock > 500)]$$
$$\equiv [(num\_orders \geq 50) \vee (num\_instock \leq 300)] \wedge$$
$$[[(num\_orders < 50) \vee (num\_orders \geq 75)] \vee (num\_instock \leq 500)]$$

Comment

Step 4 of 4

Replace $\vee$ by or and $\wedge$ by and.

$$\equiv [(num\_orders \geq 50) \text{ or } (num\_instock \leq 300)] \text{ and}$$
$$[[(num\_orders < 50) \text{ or } (num\_orders \geq 75)] \text{ or } (num\_instock \leq 500)]$$

Therefore, the negation of the statement is:

$$\boxed{\begin{array}{l}[(num\_orders \geq 50) \text{ or } (num\_instock \leq 300)] \text{ and} \\ [[(num\_orders < 50) \text{ or } (num\_orders \geq 75)] \text{ or } (num\_instock \leq 500)]\end{array}}$$

Ch 2.1

2.1.42

Consider the statement form,

$((\sim p \wedge q) \wedge (q \wedge r)) \wedge \sim q$

The objective is to use truth table to establish this statement form is either tautology or contradiction.

Comment

Step 2 of 3

Here tautology means always true and is denoted as t, and contradiction which is always false and is denoted as c.

The truth table that shows the statement form is,

p	q	r	$\sim p$	$\sim q$	$\sim p \wedge q$ 6	$q \wedge r$ 7	$(\sim p \wedge q) \wedge (q \wedge r)$ 8	$8 \wedge \sim q$
T	T	T	F	F	F	T	F	F
T	T	F	F	F	F	F	F	F
T	F	T	F	T	F	F	F	F
T	F	F	F	T	F	F	F	F
F	T	T	T	F	T	T	T	F
F	T	F	T	F	T	F	F	F
F	F	T	T	T	F	F	F	F
F	F	F	T	T	F	F	F	F

Comment

Step 3 of 3

From the above truth table, final column truth values are always false.

Hence, the statement form $((\sim p \wedge q) \wedge (q \wedge r)) \wedge \sim q$ is a contradiction.

2.1.45

(a)

Consider the statements,

B: Bob is a double math and computer science major

C: Bob is a math major

A: Ann is a math major

D: Ann is a double math and computer science major

The objective is to determine whether the statements in both (a) and (b) are logically equivalent.

Bob is a double math and computer science major and Ann is a math major, but Ann is not a double math and computer science major.

This statement can be interpreted as $(B \wedge A) \wedge \sim D$.

Comment

Step 2 of 4

(b)

Consider the statement,

Ann is a math major and Bob is a double math and computer science major.

But it is not the case that both Ann and Bob are double math and computer science majors.

This statement can be interpreted as $\sim(B \wedge D) \wedge (A \wedge B)$.

Logical equivalence of both the statements in (a) and (b):

Use De Morgan's law; write the above statement $\sim(B \wedge D) \wedge (A \wedge B)$ as,

$\sim(B \wedge D) \wedge (A \wedge B) \equiv (\sim B \vee \sim D) \wedge (A \wedge B)$ $(\text{Since } \square\, (A \wedge B) = \square\, A \vee \square\, B)$

Use the distributive law, $\equiv \{\sim B \wedge (A \wedge B)\} \vee \{\sim D \wedge (A \wedge B)\}$

Use commutative law, $\equiv \{\sim B \wedge (B \wedge A)\} \vee \{\sim D \wedge (A \wedge B)\}$

Use associative law, $\equiv \{(\sim B \wedge B) \wedge A\} \vee \{\sim D \wedge (A \wedge B)\}$

Use the negation law, $\equiv \{\mathbf{c} \wedge A\} \vee \{\sim D \wedge (A \wedge B)\}$ $(\because \square\, A \wedge A = c)$

Use the universal bound law, $\equiv \mathbf{c} \vee \{\sim D \wedge (A \wedge B)\}$

Comments (3)

Step 4 of 4

Again use the universal bound law, $\equiv \sim D \wedge (A \wedge B)$

Use commutative law twice, $\equiv (A \wedge B) \wedge \square\, D \equiv (B \wedge A) \wedge \sim D$

Hence, $\sim(B \wedge D) \wedge (A \wedge B) \equiv (B \wedge A) \wedge \sim D$.

This is nothing but the statement in (a).

Therefore, the statements (a) and (b) are equivalent.

Ch 2.1

2.1.49

The objective is to write a reason for each step in the logical equivalence provided.

The first step of the logical equivalence is:

$$(p \vee \sim q) \wedge (\sim p \vee \sim q) \equiv (\sim q \vee p) \wedge (\sim q \vee \sim p)$$

The step is obtained by using commutative law: $p \vee q \equiv q \vee p$.

Therefore, the first blank in the logical equivalence should be filled with:

(a) commutative law for $\vee$.

Consider the second step of the equivalence:

$$(p \vee \sim q) \wedge (\sim p \vee \sim q) \equiv \sim q \vee (p \wedge \sim p)$$

The step is obtained by using distributive law: $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$.

Therefore, the second blank in the logical equivalence should be filled with:

(b) distributive law.

Comment

Step 2 of 2

For the third step of the equivalence:

$$(p \vee \sim q) \wedge (\sim p \vee \sim q) \equiv \sim q \vee \mathbf{c}$$

The step is obtained by using negation law for $\wedge$: $p \wedge \sim p \equiv \mathbf{c}$.

Therefore, the third blank in the logical equivalence should be filled with:

(c) negation law for $\wedge$.

For the final step of the equivalence:

$$(p \vee \sim q) \wedge (\sim p \vee \sim q) \equiv \sim q$$

The step is obtained by using identity law for $\vee$: $p \wedge \mathbf{c} \equiv p$.

Therefore, the final blank in the logical equivalence should be filled with:

(d) identity law for $\vee$.

Thus, the final result is $(p \vee \sim q) \wedge (\sim p \vee \sim q) \equiv \sim q$.

Ch 2.1

2.1.52

The objective is to verify the following logical equivalence:

$$\sim(p \vee \sim q) \vee (\sim p \wedge \sim q) \equiv \sim p.$$

Also, mention the appropriate laws that have been used to prove that.

Comment

Step 2 of 2

Consider the left-hand side of the above equivalence as,

$$\begin{aligned}
\sim(p \vee \sim q) \vee (\sim p \wedge \sim q) &\equiv (\sim p \wedge \sim(\sim q)) \vee (\sim p \wedge \sim q) && \text{(By De Morgan's law)} \\
&\equiv (\sim p \wedge q) \vee (\sim p \wedge \sim q) && \text{(By Double negative law)} \\
&\equiv \sim p \wedge (q \vee \sim q) && \text{(By Distributive law)} \\
&\equiv \sim p \wedge (\mathbf{t}) && \text{(By Negation law for } \vee\text{)} \\
&\equiv \sim p && \text{(By Identity law for } \wedge\text{)}
\end{aligned}$$

Hence, the logical equivalence $\boxed{\sim(p \vee \sim q) \vee (\sim p \wedge \sim q) \equiv \sim p}$ has been proved.

Ch 2.1

2.1.54

Consider the following logical equivalence:

$(p\wedge(\sim(\sim p\vee q)))\vee(p\wedge q)\equiv p$

Verify the logical equivalence and supply a reason for every step.

Comment

Step 2 of 2

Use the laws of logical equivalence, replace sections of the statement form on the left $(p\wedge(\sim(\sim p\vee q)))\vee(p\wedge q)$ by logically equivalent expressions with step by step explanation until you obtain the statement form on the right as follows:

$(p\wedge(\sim(\sim p\vee q)))\vee(p\wedge q)\equiv(p\wedge(\sim(\sim p)\wedge\sim q))\vee(p\wedge q)$ by De Morgan's laws

$(p\wedge(\sim(\sim p)\wedge\sim q))\vee(p\wedge q)\equiv(p\wedge(p\wedge\sim q))\vee(p\wedge q)$ by the double negative law,

$(p\wedge(p\wedge\sim q))\vee(p\wedge q)\equiv((p\wedge p)\wedge\sim q)\vee(p\wedge q)$ by the associative laws for $\wedge$

$((p\wedge p)\wedge\sim q)\vee(p\wedge q)\equiv(p\wedge\sim q)\vee(p\wedge q)$ by the idempotent laws $p\wedge p=p$

$(p\wedge\sim q)\vee(p\wedge q)\equiv p\wedge(\sim q\vee q)$ by the distributive laws

$p\wedge(q\vee\sim q)\equiv p\wedge\mathbf{t}$ by the negation law $q\vee\sim q\equiv t$

$\equiv p$ by the identity law.

Therefore, $(p\wedge(\sim(\sim p\vee q)))\vee(p\wedge q)\equiv p$.

Ch2.2

2.2.2

Consider *p* and *q* be two statements,

p is necessary condition for *q*. *p* is called the hypothesis and *q* is called conclusion means "if not *p* then not *q*".

The given statement is, 'Fix my ceiling or I won't pay my rent'.

Consider *p*: Fix my ceiling and *q*: I won't pay my rent.

Here, the necessary condition is *p*, Fix my ceiling.

Therefore, the statement in if-then form is,

If you do not fix my ceiling, then I won't pay my rent.

2.2.12

Consider the following statement as,

"If $x > 2$ or $x < -2$ then $x^2 > 4$" for some fixed real number x.

The objective is to rewrite the above statement using the logical equivalence $(p \vee q) \to r \equiv (p \to r) \wedge (q \to r)$.

Comment

Step 2 of 2

Compare the left-hand side of the above logical equivalence with the given statement.

Then,

p: $x > 2$

q: $x < -2$

r: $x^2 > 4$

Now, use the above notations and the right-hand side of the logical equivalence to rewrite the given statement as,

"If $x > 2$ then $x^2 > 4$, and if $x < -2$ then $x^2 > 4$".

Ch2.2

2.2.14.a

(a)

To show, $p \to q \vee r$, $p \wedge \neg q \to r$ and $p \wedge \neg r \to q$ are all logically equivalent, first take

$p \to q \vee r$

by using the logical equivalence, $p \to q \equiv \neg p \vee q$

$p \to q \vee r \equiv \neg p \vee (q \vee r)$ ...(1)

Now,

$p \wedge \neg q \to r \equiv \neg(p \wedge \neg q) \vee r$

$\equiv (\neg p \vee \neg(\neg q)) \vee r$ (De Morgan's law)

$\equiv (\neg p \vee q) \vee r$ (Double negative law)

$\equiv \neg p \vee (q \vee r)$ (Associative law) ...(2)

Comments (1)

Step 2 of 4

Again,

$p \wedge \neg r \to q \equiv \neg(p \wedge \neg r) \vee q$

$\equiv (\neg p \vee \neg(\neg r)) \vee q$ (De Morgan's law)

$\equiv (\neg p \vee r) \vee q$ (Double negative law)

$\equiv \neg p \vee (r \vee q)$ (Associative law)

$\equiv \neg p \vee (q \vee r)$ (Commutative law) ...(3)

Now from (1), (2) and (3), it is clear that $p \to q \vee r$, $p \wedge \neg q \to r$ and $p \wedge \neg r \to q$ are all logically equivalent to the same statement form $\neg p \vee (q \vee r)$, i.e.,

$p \to q \vee r \equiv p \wedge \neg q \to r \equiv p \wedge \neg r \to q$

Thus, they are all logically equivalent.

Ch2.2

2.2.14.b

(b)

The statement is "if *n* is prime, then *n* is odd or *n* is 2".

Let

p = "*n* is prime"

q = "*n* is odd"

r = "*n* is 2"

Then, the statement in symbolic form is:

$$p \to q \vee r$$

Comment

Step 4 of 4

Now, the statement "if *n* is prime, then *n* is odd or *n* is 2" can be written in two different ways by using logical equivalences established in part (a), i.e.,

1. $p \wedge \neg q \to r$, i.e., If n is prime and n is not odd, then n is 2.
2. $p \wedge \neg r \to q$, i.e., If n is prime and n is not 2, then n is odd.

Ch2.2

2.2.17

Consider the two statements as,

A:"If 2 is a factor of n and 3 is a factor of n, then 6 is a factor of n."

B:" 2 is not a factor of n or 3 is not a factor of n or 6 is a factor of n."

The objective is to write this statement in symbolic form.

And identify whether these two statements are logically equivalent or not.

Also, write the truth table and explain briefly.

Comment

Step 2 of 4

Let the sentences can be written as,

P: 2 is a factor of n.

Q: 3 is a factor of n.

R: 6 is a factor of n.

$\sim P$: 2 is not a factor of n.

$\sim Q$: 3 is not a factor of n.

The symbolic forms of the above statements are as follows,

$A: (P \wedge Q) \rightarrow R$

$B: (\sim P \vee \sim Q) \vee R$

The statements A and B are logically equivalent.

Ch2.2

Proof of the equivalence,

$$A \equiv (P \wedge Q) \rightarrow R$$
$$\equiv \sim [P \wedge Q] \vee R \quad \text{(by law of implication)} \quad (\because p \rightarrow q \equiv \sim p \vee q)$$
$$\equiv (\sim P \vee \sim Q) \vee R \quad \text{(by de morgan law)}$$
$$\equiv B$$

Therefore, both the statements are equivalent.

Comment

Step 4 of 4

Consider the truth table:

P	Q	R	$P \wedge Q$	$\sim P \vee \sim Q$	A $(P \wedge Q) \rightarrow R$	B $(\sim P \vee \sim Q) \vee R$
T	T	T	T	F	T	T
T	T	F	T	F	F	F
T	F	T	F	T	T	T
T	F	F	F	T	T	T
F	F	T	F	T	T	T
F	T	F	F	T	T	T
F	T	T	F	T	T	T
F	F	F	F	T	T	T

From the truth table, it is clear that, both the formulas have identical truth values.

Therefore, the two statements are equivalent.

Thus, $\boxed{(P \wedge Q) \rightarrow R \equiv (\sim P \vee \sim Q) \vee R}$.

Ch2.2

2.2.30

The truth table for $p\wedge(q\vee r)\leftrightarrow(p\wedge q)\vee(p\wedge r)$ is shown below:

p	q	r	$p\wedge q$ A	$p\wedge r$ B	$q\vee r$ C	$p\wedge(C)$ D	$A\vee B$ E	$D\leftrightarrow E$
T	T	T	T	T	T	T	T	T
T	T	F	T	F	T	T	T	T
T	F	T	F	T	T	T	T	T
T	F	F	F	F	F	F	F	T
F	T	T	F	F	T	F	F	T
F	T	F	F	F	T	F	F	T
F	F	T	F	F	T	F	F	T
F	F	F	F	F	F	F	F	T

Comment

Step 4 of 4

From the above truth table, all values in the last column $p\wedge(q\vee r)\leftrightarrow(p\wedge q)\vee(p\wedge r)$ are true.

Hence, it is called tautology.

And both the statement forms $p\wedge(q\vee r)$ and $(p\wedge q)\vee(p\wedge r)$ are same truth values.

Hence, the statement forms $p\wedge(q\vee r)$ and $(p\wedge q)\vee(p\wedge r)$ are logically equivalent.

If the statement forms $p\wedge(q\vee r)$ and $(p\wedge q)\vee(p\wedge r)$ are logically equivalent, then the biconditional form $p\wedge(q\vee r)\leftrightarrow(p\wedge q)\vee(p\wedge r)$ is a tautology.

Hence, the result $p\wedge(q\vee r)\equiv(p\wedge q)\vee(p\wedge r)$ is verified.

Ch2.2

2.2.33

Given statement is "This integer is even if, and only if, it equals the twice some integer."

Suppose p: "This integer is even"

and q: "This integer equals the twice some integer"

Then the given statement can be symbolized as " p if , and only if, q"

The conjuction of two if-then statements can be written as

"If p then q" *and* "If q then p"

So the conjuction of two if-then statements for the given statement is

"If this integer is even, then it equals twice some integer" *and* "if the integer equals twice some integer then, this integer is even."

2.2.39

The given statement is of the form if-then.

If a security code is not entered, then this door will not open.

2.2.45

The objective is to write the following statement in if-then form:

"A necessary condition for this computer program to be correct is that it not produce error messages during translation."

Comment

Step 2 of 2

Let r and s be propositions, and r is a necessary condition for s means:

"If not r then not s"

That is $\sim r \to \sim s \equiv s \to r$.

From the statement write the propositions as:

r: Computer program does not produce error messages during translation.

s: Computer program is correct.

Write the statement using if-then as follows:

"If the computer program is correct, then it does not produce error messages during translation."

Ch2.2

2.2.47

(a)

The objective is to write the following statement without using the symbol $\rightarrow$ or $\leftrightarrow$.

$p \wedge \sim q \rightarrow r.$

Use the logical equivalence $p \rightarrow q \equiv \sim p \vee q.$

Apply the above rule to the statement $p \wedge \sim q \rightarrow r$ by considering p as $p \wedge \sim q$.

$$p \wedge \sim q \rightarrow r \equiv (p \wedge \sim q) \rightarrow r$$

$$\boxed{p \wedge \sim q \rightarrow r \equiv \sim (p \wedge \sim q) \vee r}$$

Thus, right hand side part of the above equivalence does not contain the symbol $\rightarrow$ or $\leftrightarrow$.

Comment

Step **2** of 2

(b)

The objective is to rewrite the statement using $\wedge$ and $\sim$ form.

Apply the negation law~ $\sim(\sim p) \equiv p$ to $\sim (p \wedge \sim q) \vee r.$

$$\begin{aligned} \sim (p \wedge \sim q) \vee r &\equiv \sim \left[\sim \left(\sim (p \wedge \sim q) \vee r\right)\right] \\ &\equiv \sim \left[\sim \left(\sim (p \wedge \sim q)\right) \wedge \sim r\right] && \text{since } \sim (p \vee q) \equiv \sim p \wedge \sim q \\ &\equiv \sim \left[(p \wedge \sim q) \wedge \sim r\right] && \text{since } \sim (\sim p) \equiv p \end{aligned}$$

Therefore, the statement using $\wedge$ and $\sim$ is, $p \wedge \sim q \rightarrow r \equiv \boxed{\sim \left[(p \wedge \sim q) \wedge \sim r\right]}$.

Ch2.2

2.2.48

Consider the statement,

$p \vee \sim q \rightarrow r \vee q$

The objective is to write the given statement without using the symbol $\rightarrow$ or $\leftrightarrow$.

Use the following logical equivalences:

$p \rightarrow q \equiv \sim p \vee q$

$p \leftrightarrow q \equiv (\sim p \vee q) \wedge (\sim q \vee p)$

Comment

Step 2 of 3

Take P as $p \vee \sim q$ and Q as $r \vee q$.

So the logical equivalence is,

$P \rightarrow Q \equiv \sim P \vee Q$

$(p \vee \sim q) \rightarrow (r \vee q) \equiv \sim (p \vee \sim q) \vee (r \vee q)$

$\equiv (\sim p \wedge \sim (\sim q)) \vee (r \vee q)$ De Morgan law

$\equiv (\sim p \wedge q) \vee (r \vee q)$

Hence, the statement without using the symbol $\rightarrow$ is,

$p \vee \sim q \rightarrow r \vee q \equiv \boxed{(\sim p \wedge q) \vee (r \vee q)}$.

Comment

Step 3 of 3

(b)

The objective is to rewrite the statement form using $\wedge$ and $\sim$.

Take $(\sim p \wedge q) \vee (r \vee q)$ from (a) and apply the law $u \vee v \equiv \sim (\sim u \wedge \sim v)$ with,

$u = (\sim p \wedge q)$, $v = (r \vee q)$

The given statement can be written as,

$$
\begin{aligned}
& p \vee \sim q \rightarrow r \vee q \\
& \quad \equiv (\sim p \wedge q) \vee (r \vee q) && \text{from } (a) \\
& \quad \equiv \sim [\sim (\sim p \wedge q) \wedge \sim (r \vee q)] && \text{use } u \vee v \equiv \sim (\sim u \wedge \sim v) \\
& \quad \equiv \sim [\sim (\sim p \wedge q) \wedge (\sim r \wedge \sim q)]
\end{aligned}
$$

Therefore, the equivalent statement using the symbols $\wedge$ and $\sim$ is,

$p \vee \sim q \rightarrow r \vee q \equiv \boxed{\sim [\sim (\sim p \wedge q) \wedge (\sim r \wedge \sim q)]}$.

Ch2.2

2.2.50

The objective is to write the following statement without using the symbol $\rightarrow$ or $\leftrightarrow$.

$(p \rightarrow (q \rightarrow r)) \leftrightarrow ((p \wedge q) \rightarrow r)$.

Use the logical equivalence $p \rightarrow q \equiv \sim p \vee q$.

$q \rightarrow r \equiv -q \vee r$

Then,

$p \rightarrow (q \rightarrow r) \equiv -p \vee (-q \vee r)$.

On the other hand,

$$\begin{aligned}(p \wedge q) \rightarrow r &\equiv -(p \wedge q) \vee r \\ &\equiv (-p \vee -q) \vee r\end{aligned}$$

Comment

Step 2 of 3

Use the logical equivalence $p \leftrightarrow q \equiv (-p \vee q) \wedge (-q \vee p)$ for $(-p \vee (-q \vee r)) \leftrightarrow (-p \vee -q) \vee r$.

$$\left[\begin{aligned}&\left(\underbrace{-(-p \vee (-q \vee r))}_{-r} \vee \underbrace{((-p \vee -q) \vee r)}_{s}\right) \\ &\wedge \left(\underbrace{-((-p \vee -q) \vee r)}_{-q}\right) \vee \left(\underbrace{-p \vee (-q \vee r)}_{p}\right)\end{aligned}\right]$$

$$= \left.\begin{aligned}&[-(-p) \wedge -(-q \vee r) \vee (-p \vee -q \vee r)] \\ &\wedge [-(-p \vee -q) \wedge -r \vee (-p \vee -q \vee r)]\end{aligned}\right\}$$

$$= \left.\begin{aligned}&[(p \wedge q \wedge -r) \vee (-p \vee -q \vee r)] \\ &\wedge [(p \wedge q \wedge -r) \vee (-p \vee -q \vee r)]\end{aligned}\right\}$$

$$= \boxed{((p \wedge q \wedge -r) \vee (-p \vee -q \vee r))}.$$

Comments (2)

Step 3 of 3

(b)

The objective is to rewrite the statement using $\wedge$ and $-$ form.

Apply the law $-p \vee -q \equiv -(p \wedge q)$ to $(p \wedge q \wedge -r) \vee (-p \vee -q \vee r)$.

$$\begin{aligned}(p \wedge q \wedge -r) \vee (-p \vee -q \vee r) &\equiv (p \wedge q \wedge -r) \vee (-(p \wedge q) \vee r) \\ &\equiv (p \wedge q \wedge -r) \vee -(p \wedge q \wedge -r) \\ &\equiv -(-(p \wedge q \wedge -r)) \vee -(p \wedge q \wedge -r) \\ &\equiv -[-(p \wedge q \wedge -r) \wedge (p \wedge q \wedge -r)]\end{aligned}$$

Therefore, the statement using $\wedge$ and $-$ is:

$$(p \wedge q \wedge -r) \vee (-p \vee -q \vee r) \equiv \boxed{-[-(p \wedge q \wedge -r) \wedge (p \wedge q \wedge -r)]}.$$

Ch2.3

2.3.9

Consider the argument.

$p \wedge q \to \sim r$

$p \vee \sim q$

$\sim q \to p$

$\therefore \sim r$

Construct a truth table as shown below, and indicate which columns represent the premises and which represent the conclusion.

Comment

Step 2 of 3

The truth table is as follows:

						Premises			Conclusion	
p	q	r	$\sim q$	$\sim r$	$p \wedge q$	$p \wedge q \to \sim r$	$p \vee \sim q$	$\sim q \to p$	$\sim r$	
T	T	T	F	F	T	F	T	T	F	
T	T	F	F	T	T	T	T	T	T	
T	F	T	T	F	F	T	T	T	F	(*)
T	F	F	T	T	F	T	T	T	T	
F	T	T	F	F	F	T	F	T	F	
F	T	F	F	T	F	T	F	T	T	
F	F	T	T	F	F	T	T	F	F	
F	F	F	T	T	F	T	T	F	T	

Comment

Step 3 of 3

In the above truth table, the selected row (*), shows that it is possible for an argument of this form to have true premises and a false conclusion.

Hence, this argument form is invalid.

Ch2.3

2.3.11

Consider the following argument:

$$\begin{array}{l} p \rightarrow q \vee r \\ \sim q \vee \sim r \\ \hline \therefore \sim p \vee \sim r \end{array}$$

The objective is to construct a truth table to check whether the above argument is valid or not.

Also, represent the premises and conclusion columns separately in the truth table.

Comment

Step 2 of 2

The required truth table for the given argument is as follows:

							Premises		Conclusions	
p	q	r	$\sim p$	$\sim q$	$\sim r$	$q \vee r$	$p \rightarrow q \vee r$	$\sim q \vee \sim r$	$\sim p \vee \sim r$	
T	T	T	F	F	F	T	T	F	F	
T	T	F	F	F	T	T	T	T	T	
T	F	T	F	T	F	T	T	T	F	→ (*)
T	F	F	F	T	T	F	F	T	T	
F	T	T	T	F	F	T	T	F	T	
F	T	F	T	F	T	T	T	T	T	
F	F	T	T	T	F	T	T	T	T	
F	F	F	T	T	T	F	T	T	T	

In the truth table, row (*) shows that the argument has true premises with a false conclusion, this implies the argument is invalid.

Therefore, the argument given above is **invalid.**

Ch2.3

2.3.12

Construct a truth table as shown below, and indicate which columns represent the premises and which represents the conclusion.

Premises Conclusion

p	q	$p \to q$	q	p	
T	T	T	T	T	
T	F	F	F	T	
F	T	T	T	F	→ (*)
F	F	T	F	F	

In the truth table, row (*) shows that it is possible for an argument of this form to have true premises and a false conclusion.

Thus, this form of argument is invalid.

Comment

Step 2 of 2

(b)

Construct a truth table as shown below, and indicate which columns represent the premises and which represents the conclusion.

Premises Conclusion

p	q	$p \to q$	$\neg p$	$\neg q$	
T	T	T	F	F	
T	F	F	F	T	
F	T	T	T	F	→ (*)
F	F	T	T	T	

In the truth table, row (*) shows that it is possible for an argument of this form to have true premises and a false conclusion.

Ch2.3

2.3.14

The objective is to construct a truth table for the following argument form:

p
$\therefore p \vee q$

Fill in the four possible combinations of truth values for p and q.

p	q
T	T
T	F
F	T
F	F

Then fill in the columns for $p \vee q$ using the definitions of *or*.

p	q	p	$p \vee q$
T	T	T	T
T	F	T	T
F	T	F	T
F	F	F	F

Comment

Step 2 of 2

From the table, observe that the first two columns give the assignment of truth-values, and the next two columns gives the truth values of the premises.

Label the premise and conclusion in the truth table as follows:

		Premise	Conclusion	
p	q	p	$p \vee q$	
T	T	T	T	Critical rows
T	F	T	T	Critical rows
F	T	F		
F	F	F		

Observe that in critical rows, the premise and conclusion are true.

Therefore, this form of argument is valid.

Ch2.3

2.3.41

The objective is to deduce the conclusion $\sim q$ form the following premises:

a. $\sim p \vee q \to r$
b. $s \vee \sim q$
c. $\sim t$
d. $p \to t$
e. $\sim p \wedge r \to \sim s$

Use suitable rules of inference to draw the conclusion.

Step Reason

1. $\sim t$ Premise (c)

2. $p \to t$ Premise (d)

Use Modus Tollens on steps **1** and **2.**

$p \to q$
$\sim q$
$\therefore \sim p$

Step Reason

3. $\sim p$ Modus Tollens on steps **1** and **2**

Comment

Step 2 of 4

Use generalization rule $p // \therefore p \vee q$ to step **3**.

Step Reason

4. $\sim p \vee q$ Rule of generalization on $\sim p$

5. $\sim p \vee q \to r$ Premise (a)

Use Modus Ponens on steps **4** and **5.**

$p \to q$
p
$\therefore q$

Step Reason

6. r Modus Ponens on steps **4** and **5**

Ch2.3

Use Conjunction $p, q // \therefore p \wedge q$ on steps **3** and **6**.

Step Reason

7. $\sim p \wedge r$ Conjunction on steps **3** and **6**

8. $\sim p \wedge r \rightarrow \sim s$ Premise (e)

Use Modus Ponens on steps **7** and **8**.

$p \rightarrow q$
p
$\therefore q$

Step Reason

9. $\sim s$ Modus Ponens on steps **7** and **8**

10. $s \vee \sim q$ Premise (b)

Comment

Step 4 of 4

Use Elimination $p \vee q, \sim q // \therefore p$ on steps **9** and **10.**

11. $\sim q$ By Elimination on steps **9** and **10**

Therefore, conclusion is $\boxed{\sim q}$.

Ch2.3

2.3.43

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SR: 5892

$a.\ \sim p \rightarrow r \wedge \sim s$

$b.\quad t \rightarrow s$

$c.\quad u \rightarrow \sim p$

$d.\quad \sim w$

$e.\quad u \vee w$

$f.\quad \therefore \sim t$

To check the validity of this argument, we proceed as

$d. \sim w$

$e.\quad u \vee w$

$\therefore\ u$ ---(1) from elimination.

Comment

Step 2 of 3

(1) $c. \Rightarrow \sim p$ is true---(2) from modus ponens

(2), $a.\ \Rightarrow r \wedge \sim s$ is true ---(3) from modus ponens

Comment

Step 3 of 3

(3) $\Rightarrow \therefore r\ ,\ \therefore \sim s$ ---(4) from specialization

(4), $b.\ \Rightarrow \sim t$ is true.

$\therefore \sim t$ from modus tollens

So the conclusion f. is valid.

Ch2.3

2.3.44

(1) $\square s \to \square t$ by premise (c)

$\square s$ by premise (e)

$\therefore \square t$ by modus ponens

Comment

Step 2 of 8

(2) $w \vee t$ by premise (g)

$\square t$ by (1)

$\therefore w$ by elimination

Comment

Step 3 of 8

(3) $\square q \vee s$ by premise (d)

$\square s$ by premise (e)

$\therefore \square q$ by elimination

Comment

Step 4 of 8

(4) $p \to q$ by premise (a)

$\square q$ by (3)

$\therefore \square p$ by modus tollens

Ch2.3

Step 5 of 8

(5) $r \vee s$ by premise (b)

□ s by premise (e)

$\therefore r$ by elimination

Comment

Step 6 of 8

(6) □ p by (4)

r by (5)

$\therefore$□ $p \wedge r$ by conjunction

Comment

Step 7 of 8

(7) □ $p \wedge r \rightarrow u$ by premise (f)

□ $p \wedge r$ by (6)

$\therefore u$ by modus ponens

Comment

Step 8 of 8

(8) w by (2)

u by (7)

$\therefore u \wedge w$ by conjunction